Fall 2012 MAT 334 Exam 1

You have 50 minutes. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. *Let C denote the bottom half of the positively oriented unit circle with center i, i.e., counterclockwise from $i-1$ to $i+1$. Evaluate*

$$\int_C (z-i)^{-3} dz.$$

Problem 2. *Let D_1 and D_2 be open subsets of $\mathbb{C}$. Prove that their intersection, $D_1 \cap D_2$, is also open.*

Problem 3. *Find all solutions z of the equation*

$$z^5 = \sqrt{3} - i.$$

Problem 4. *Consider the following limit:*

$$\lim_{z \to 2} \frac{\bar{z} - 2}{z - 2}.$$

If the limit exists, prove it (using δ-ϵ or some other rigorous means). If the limit does not exist, show why not.

Problem 5. *For which $z \in \mathbb{C}$ does the series*

$$\sum_{n=0}^{\infty} |z^n + z^{n+1}|$$

converge?