Fall 2012 MAT 334 Exam 2

You have 50 minutes. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. *Let the curve γ be the positively-oriented boundary of the square with corners at $2 + 2i$, $-2 + 2i$, $-2 - 2i$, and $2 - 2i$. Evaluate the following:*

$$\int_\gamma \frac{e^z}{z^2 + 2z - 3}dz.$$

Problem 2. *For what $z \in \mathbb{C}$ is the function $f(z) = |z|^2$ (complex) differentiable? For what $z \in \mathbb{C}$ is f not differentiable?*

Problem 3. *Suppose f and g are holomorphic functions such that:*

- *f has a zero of order $m > 0$ at $z_0 \in \mathbb{C}$.*

- *g has a pole of order m at z_0.*

Prove that $f \cdot g$ can be analytically extended to a nonzero value at z_0.

Problem 4. *Let*
$$f(z) = \frac{(z + 2)\sin z}{(e^z - 1)^2}.$$

Show that f has a pole of order 1 at $z = 0$, and then compute $\operatorname{Res}(f; 0)$.

Problem 5. *Let γ be any smooth piecewise continuous curve from $-i$ to i that does not pass through the* positive real axis $[0, \infty)$. *Compute*

$$\int_\gamma \frac{1}{z}dz.$$

*(Hint: The answer is **not** $+\pi i$).*