

MAT 334 Practice Exam 1

You have 50 minutes. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. Let L denote the line segment from $3 - 2i$ to -1 . Evaluate

$$\int_L z dz.$$

Problem 2. Prove or disprove: if both $U \subseteq \mathbb{C}$ and $V \subseteq U$ are open, then

$$U \setminus V = \{z \mid z \in U \text{ and } z \notin V\},$$

i.e., the set of all points in U which are not in V , is also open.

Problem 3. Find all values of 2^{-i} .

Problem 4. Does the limit

$$\lim_{z \rightarrow 0} e^{\frac{1}{z}}$$

exist? Justify your answer. (Note: ∞ is allowed to be a possible limit value.)

Problem 5. (Fall 2012, Midterm 1) Consider, for any fixed $z \in \mathbb{C}$, the series

$$\sum_{n=0}^{\infty} |z^n + z^{n+1}|.$$

For which z does this series converge? Diverge?