Spring 2014 MAT 336 Practice Exam 1

You have 1 hour. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. *Does the series*

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^4 + 4}}$$

converge? Justify your answer.

Problem 2. *Find* $\liminf_n x_n$, *where*

$$x_n = \left(2 + \frac{1}{n}\right)\cos\frac{\pi n}{6}.$$

Problem 3. *Let* $d : \mathbb{R} \times \mathbb{R} \to [0, \infty)$ *be defined by*

$$d(x, y) = (x - y)^2.$$

Is d a metric on $\mathbb{R}$? Why or why not?

Problem 4. *Show that the set*

$$(-1, 0) \cup (0, 1) = \{x \in \mathbb{R} \mid -1 < x < 0 \text{ or } 0 < x < 1\}$$

has the same cardinality as $\mathbb{R}$.

Problem 5. *Let (a_n) be a sequence of real numbers, and suppose $\lim_n a_n = L$. Show the following limit also holds:*

$$\lim_n \frac{1}{n}(a_1 + a_2 + \cdots + a_n) = L.$$

Extra Problem. *Give an example of a sequence (x_n) in $\mathbb{R}$ such that*

$$\lim_n |x_{n+1} - x_n| = 0,$$

but (x_n) does not converge to a real number.