

Spring 2014 MAT 336 Practice Exam 2

You have 1 hour. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. Show that \mathbb{Q} is not a connected subset of \mathbb{R} .

Problem 2. Suppose U and V are open subsets of \mathbb{R} . Show that

$$U \times V = \{(x, y) \in \mathbb{R}^2 \mid x \in U, y \in V\}$$

is an open subset of \mathbb{R}^2 .

Problem 3. Suppose $f : (-1, 1) \rightarrow \mathbb{R}$ is differentiable at 0, and suppose $f'(0) > 0$. Show that there is some $x \in (0, 1)$ such that $f(x) > f(0)$.

Problem 4. Show that $1 - \cos x \leq x \sin x$ for all $x \in [0, \frac{\pi}{2}]$. Hint: Apply the mean value theorem, and note that $\sin x$ is monotone on $[0, \frac{\pi}{2}]$.

Problem 5. Suppose $f : [0, 1] \rightarrow [0, 1]$ is continuous. Show that f has a fixed point, i.e., there is some $x \in [0, 1]$ such that $f(x) = x$.