Spring 2014 MAT 336 Exam 1

You have 1 hour. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. *Consider the set*

$$A = \{(1 + x^2)^{-1} \mid x \in \mathbb{R}\}.$$

Find $\sup A$ *and* $\inf A$*, and justify your answers.*

Problem 2. *Find the following limit:*

$$\lim_n [\log(n + 3) - \log n].$$

Prove that your answer is correct.

Problem 3. *Give a specific example of a sequence* (x_n) *in* $\mathbb{R}$ *and a subsequence* (x_{n_k}) *of* (x_n)*, such that* $\sum_{n=0}^{\infty} x_n$ *converges, but* $\sum_{k=0}^{\infty} x_{n_k}$ *diverges.*

Problem 4. *Let* d *be the metric on* $X = [1, \infty)$ *given by*

$$d(x, y) = \left| \frac{1}{x} - \frac{1}{y} \right|.$$

(You do NOT have to show d is a metric.) *Show that* (X, d) *is* not *complete.*

Problem 5. *Let* (x_n) *be a bounded sequence in* $\mathbb{R}$*, and let* $\ell = \limsup_n x_n$*. Show that there exists a subsequence* (x_{n_k}) *that converges to* ℓ*.*