Spring 2014 MAT 336 Exam 2

You have 1 hour. Answer 4 of the following 5 questions. If you answer all 5, your score will be determined by the best 4 solutions you provide.

Problem 1. *Give an example of a continuous function* $f : \mathbb{R}^2 \to \mathbb{R}$ *and an open subset* $U \subseteq \mathbb{R}^2$ *such that* $f(U) = \{f(x) \mid x \in \mathbb{R}^2\}$ *is not open.*

Problem 2. *Show that* $\ln(1 + x) \leq x$ *for any* $x \geq 0$. Hint: One possibility is to find a way to apply the mean value theorem.

Problem 3. *Let* $f : \mathbb{R} \to \mathbb{R}$ *be continuous, and suppose* f *is periodic, that is, there is some* $d > 0$ *such that* $f(x + d) = f(x)$ *for all* $x \in \mathbb{R}$*. Show that* f *achieves its maximum value (more specifically, show there is some* $x_0 \in \mathbb{R}$ *such that* $f(x_0) \geq f(x)$ *for all* $x \in \mathbb{R}$*).*

Problem 4. *Suppose* $f : \mathbb{R} \to \mathbb{R}$ *satisfies that*

$$|f(x) - f(y)| \leq (x - y)^2$$

for all $x, y \in \mathbb{R}$*. Prove that* f *is a constant function.*

Problem 5. *Consider the set* $A = \{(x, \sin \frac{1}{x}) \in \mathbb{R}^2 \mid x > 0\} \subseteq \mathbb{R}^2$*. Find all the limit points of* A*.*